The Discount for Lack of Marketability: Quantifying the Risk of Illiquidity

Mark L. Zyla CPA/ABV, CFA, ASA
Managing Director
Acuitas, Inc.
Atlanta, Georgia
Overview

- Do investors consider the risk of illiquidity in measuring the value of various assets, particularly equities?
  - If so, how much impact does liquidity have on the overall value of the asset as determined by the market?
  - Conversely, how much of a discount to value if an asset (say, a minority interest in a private company) is considered relatively illiquid?

- How does one measure the premium investors pay for liquidity? Can the inverse be used as a measure of discount for lack of marketability?
What We Do Not Know: 10 Unsolved Problems in Finance

8. What is the value of liquidity?

- Value of corporations holding cash is more than forgone interest.
- Spread between corporate bonds and treasuries is more than just default risk.
- Private equity firms price investment in private companies as if publicly traded—how does one model the liquidity difference?

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  - Conversely, how much of a discount to value if an asset (say, a minority interest in a private company) is considered relatively illiquid?

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Do Investors Value Liquidity?

- Transaction costs on publicly traded equities.
  - Bid ask spread
  - Price impact of trade
  - Opportunity Cost
  - Commission

Damodaran, Aswath; *Marketability and Value: Measuring the Illiquidity Discount*; July 2005; http://pages.stern.nyu.edu/~adamodar/
Financial Crisis and Liquidity

- Long Term Capital Management
  - Bet on the yield differential between the 30 year and 29 year U.S. Treasury Bonds
  - Long position in the 29 year bond; short position in the 30 year bond which had a 5 basis point differential- essentially betting that the liquidity difference would drop.
  - However Russia defaulted on bonds- yield spread to 35 basis points
It’s a Wonderful Life

Definitions

- Liquidity- the ability to quickly convert property to cash or pay a liability.
- Marketability- the ability to quickly convert property to cash at minimal cost.

International Glossary of Business Valuation Terms. (See AICPA SSVS No.1 and American Society of Appraisers Business Valuation Standards among others.)
Where does the risk of illiquidity reside?

- Most valuation experts measure the fair market value of an interest in a private company through valuation techniques which measure the company as if it were publicly traded. Discounted Cash Flow Analysis and Guideline Public Company Analysis.

- The risk of illiquidity or lack of marketability is at the security not the company level. Consequently, risk modeled through the DCF and GPGM typically does not include risk of illiquidity.

- Creates a need to measure the impact of risk of lack of liquidity upon value of the interest.
Proxies to Measure the Discount for Lack of Marketability (Liquidity)

- Restricted Stock Studies
- Pre-Initial Public Offering ("IPO") Studies
- Hypothetical Put Option
- Differential Cash Flow
- Discount Rate Adjustment
Restricted Stock Studies

- **Advantages**
  - Compares the price of a security in a private placement to its publicly traded counterpart.
    - The only difference in price theoretically is marketability.
  - Studies have been one of the more frequently referenced source of empirical data for proxy for discount for lack of marketability.
  - Internal Revenue Service Revenue Ruling 77-287 *Valuation of Securities Restricted from Immediate Resale* references studies as empirical evidence of lack of marketability.

- **Disadvantages**
  - Restricted shares often differ than their publicly traded counterparts.
    - Different holding period.
    - Empirical data often shows a wide range.
    - Limited information about the terms of private placement itself.
    - Data is becoming stale.
### Restricted Stock Studies
**Summary of Observed Price Discounts**

<table>
<thead>
<tr>
<th>Restricted Stock Study</th>
<th>Observation Period of Study</th>
<th>Observed Price Discount</th>
<th>Average Price Discount</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEC Overall Average</td>
<td>1966–69</td>
<td>25.8 percent</td>
<td></td>
</tr>
<tr>
<td>SEC Nonreporting OTC Companies</td>
<td>1966–69</td>
<td>32.6 percent</td>
<td></td>
</tr>
<tr>
<td>Milton Gelman</td>
<td>1968–70</td>
<td>33.0 percent</td>
<td></td>
</tr>
<tr>
<td>Robert R. Trout</td>
<td>1968–72</td>
<td>33.5 percent</td>
<td></td>
</tr>
<tr>
<td>Robert E. Moroney</td>
<td>1969–72</td>
<td>35.6 percent</td>
<td></td>
</tr>
<tr>
<td>J. Michael Maher</td>
<td>1968–73</td>
<td>35.4 percent</td>
<td></td>
</tr>
<tr>
<td>Standard Research Consultants</td>
<td>1978–82</td>
<td>45.0 percent</td>
<td></td>
</tr>
<tr>
<td>Willamette Management Associates</td>
<td>1981–84</td>
<td>31.2 percent</td>
<td></td>
</tr>
<tr>
<td>Hertzel &amp; Smith</td>
<td>1980–87</td>
<td>13.5 percent</td>
<td></td>
</tr>
<tr>
<td>William L. Silber</td>
<td>1981–88</td>
<td>33.8 percent</td>
<td></td>
</tr>
<tr>
<td>Baja, Denis, Ferris, and Sarin [a]</td>
<td>1990–95</td>
<td>22.2 percent</td>
<td></td>
</tr>
<tr>
<td>Johnson Study</td>
<td>1991–95</td>
<td>20.0 percent</td>
<td></td>
</tr>
<tr>
<td>Management Planning, Inc.</td>
<td>1980–96</td>
<td>27.1 percent</td>
<td></td>
</tr>
<tr>
<td>FMV Opinions, Inc.[b]</td>
<td>1980–97</td>
<td>23.0 percent</td>
<td></td>
</tr>
<tr>
<td>Columbia Financial Advisors, Inc.</td>
<td>1996–97</td>
<td>21.0 percent</td>
<td></td>
</tr>
<tr>
<td>Columbia Financial Advisors, Inc.</td>
<td>1997–98</td>
<td>13.0 percent</td>
<td></td>
</tr>
<tr>
<td>Hall and Polacek</td>
<td>1979–92</td>
<td>23.0 percent</td>
<td></td>
</tr>
<tr>
<td>Stryker and Pittock</td>
<td>1978–82</td>
<td>45.0 percent</td>
<td></td>
</tr>
<tr>
<td>Trugman Valuation Associates</td>
<td>2007–08</td>
<td>18.1 percent</td>
<td></td>
</tr>
<tr>
<td>Trugman Valuation Associates</td>
<td>2009–10</td>
<td>16.6 percent</td>
<td></td>
</tr>
<tr>
<td>LiquiStat</td>
<td>2005–06</td>
<td>32.8 percent</td>
<td></td>
</tr>
</tbody>
</table>

[a] This study attributes price discount to factors other than marketability (i.e. compensation for the cost of assessing the quality of the firm and for the anticipated costs of monitoring the future decisions of its managers).

Pre-IPO Studies

- Advantages
  - Pre-IPO studies compare the price in private transaction of a company's stock prior to going public to its price in an IPO.

- Disadvantages
  - Transactions are typically with insiders which may not be reflective of fair market value.
  - Private transactions in the study are typically not adjusted for time value of money, changes within the corporation itself.
## Pre-IPO Studies (Emory)

<table>
<thead>
<tr>
<th>Study</th>
<th>Prospectuses Reviewed</th>
<th>Qualifying Transactions</th>
<th>Indicated Discount</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>1980–1981</td>
<td>97</td>
<td>13</td>
<td>60%</td>
</tr>
<tr>
<td>1985–1986</td>
<td>130</td>
<td>21</td>
<td>43%</td>
</tr>
<tr>
<td>1987–1989</td>
<td>98</td>
<td>27</td>
<td>45%</td>
</tr>
<tr>
<td>1989–1990</td>
<td>157</td>
<td>23</td>
<td>45%</td>
</tr>
<tr>
<td>1990–1991</td>
<td>266</td>
<td>35</td>
<td>42%</td>
</tr>
<tr>
<td>1992–1993</td>
<td>443</td>
<td>54</td>
<td>45%</td>
</tr>
<tr>
<td>1994–1995</td>
<td>318</td>
<td>46</td>
<td>45%</td>
</tr>
<tr>
<td>1995-1997</td>
<td>732</td>
<td>91</td>
<td>43%</td>
</tr>
<tr>
<td>1997-2000</td>
<td>1,847</td>
<td>283</td>
<td>50%</td>
</tr>
<tr>
<td>Overall</td>
<td>4,088</td>
<td>593</td>
<td>47%</td>
</tr>
</tbody>
</table>
Hypothetical Put Option

- **Chaffe Put Option Model**
  - Uses a Black-Scholes Option Pricing Model to create a synthetic put option.
  - One issue is that if applied to a liquid stock it would still indicate a discount is warranted.
  - Testing against restricted stocks indicates that at lower volatilities the model compares favorably to empirical data but diverges as volatilities increase (>50%).

- **Longstaff Lookback Put Option Model**
  - Sets an upper boundary of the discount through a lookback method.
  - Assumes a special market timing ability

- **Finnerty Average Strike Put Option**
  - Finnerty uses average strike put option (Asian Put Option).
  - Eliminates special market timing assumption of Longstaff
  - Works well for lower volatility and lower time horizon holding periods.

- **Ghaidarov Average Strike Put Option**
  - Initially developed as a critique of the Finnerty Model
Hypothetical Put Option References


Differential Cash Flow

- QMDM- Quantitative Marketability Discount Model
  - Developed by Chris Mercer of Mercer Capital Management which was first presented in 1998.
  - Does not directly compute DLOM but value of subject interest using cash flow unique to the interest. The implied DLOM is derived by comparing to equity as a whole.
  - Four unique assumptions in the discounted cash flow of the subject interest.
    - Holding period
      - Judgment based upon facts and circumstance
    - Expected cash flow to the shareholder
      - Dividends or other distributions during the holding period
    - Value at the end of the holding period
      - Estimated through use of Gordon- Growth Model
    - Holding period return.
      - WACC or Equity (typically equity) plus holding period premium
  - Application of the liquidity theory where the value of an asset is based on the present value of future dividends reduced for factors such as transaction costs, economic rent and other non-liquidity risk factors.

Discount Rate Adjustment

- Meulbroake CAPM
  - Developed to estimate the cost of holding a single stock in retirement plan compared to a diversified portfolio.
  - Theory is DLOM is the cost of holding a single security compared to the diversified portfolio.
  - Compares specific company returns versus the CAPM which assumes a diversified portfolio.
  - Tests indicate that the model works at low volatilities but unusual results at higher volatilities.

- Tabak CAPM
  - Similar in theory as the Meulbroek model except that Tabak uses variance of company return to variance of market return rather than standard deviation of returns.
  - Testing of the model indicates discounts that appear unreasonable in many circumstances (> 100%), therefore not recommended.

Mandelbaum Factor Analysis

Begin with Private versus Public sales of stock (average 35% from Restricted Stock Studies and 45% from Pre-IPO studies).

Then compare subject interest through:

- Financial statement analysis
- Dividend policy
- History and nature of the company
- Management
- Control if any
- Stock restrictions
- Holding period
- Redemption policy
- Costs of IPO

Mandelbaum v. Commissioner (TC Memo 1995-225)
Conclusions

- There is evidence that investors prefer liquidity and include a premium for liquidity in valuations. Non liquid assets such as equities in private companies have additional risk for lack of marketability.
- The direct measurement of illiquidity impact is difficult from empirical data.
- Valuation analysts use various studies and models as proxies for discounts for lack of marketability
  - Restricted Stock
  - Pre-IPO
  - Hypothetical Put Option
  - QMDM
  - Discount rate differentials
- Compare the subject interest to the empirical data when selecting an appropriate discount.
- Use more than one methodology, if appropriate.
Mark Zyla, CPA/ABV, CFA, ASA

Mark is a Managing Director of Acuitas, Inc. an Atlanta, Georgia based valuation and litigation consultancy firm.

Mark received a BBA degree in Finance from the University of Texas at Austin and an MBA degree with a concentration in Finance from Georgia State University. Mark also completed the Mergers and Acquisitions Program at the Aresty Institute of The Wharton School of the University of Pennsylvania and the Valuation Program at the Graduate School of Business at Harvard University. He is a Certified Public Accountant, Accredited in Business Valuation (“CPA/ABV”), Certified in Financial Forensics (“CFF”) by the AICPA, a Chartered Financial Analyst (“CFA”), and an Accredited Senior Appraiser with the American Society of Appraisers certified in Business Valuation (“ASA”).

Mark is a former member of the Business Valuations Committee of the AICPA, and a former Chairman of the ABV Examination Committee of the AICPA. He is also a member of the Business Valuation Committee of the ASA where he serves on the Standards Subcommittee. He is one of the authors of the International Glossary of Business Valuation Terms which has been adopted by the major valuation organizations. He is also a member of the Liabilities Working Group of the International Valuation Standards Council (“IVSC”). Mark is on the Advisory Council of the Master of Science in Finance program at the University of Texas at Austin. In 2013, Mark was inducted into the AICPA Business Valuation Hall of Fame.

Mark is a frequent presenter and author on valuation issues. He has taught valuation courses at the FBI Academy in Quantico, Virginia and to the PCAOB in Washington D.C. He is on the faculty of the Federal Judicial Center and the National Judicial College teaching business valuation concepts to judges. Mark is author of Fair Value Measurement: Practical Guidance and Implementation 2nd ed. published by John Wiley & Sons, Inc. (2013). Mark is also the co-author of the course, “Fair Value Accounting: A Critical New Skill for All CPAs” published by the AICPA.
Mark Zyla, CPA/ABV, CFA, ASA

Managing Director
Acuitas Inc.
One Midtown Plaza
1360 Peachtree Street NE, Suite 950
Atlanta, GA  30309
404-898-1137
www.acuitasinc.com
Fair Value Measurements Blog:
www.acuitasinc.com/blog